Nonconventional Transmission Zeros in Distributed Rectangular Structures

BRUNO BIANCO AND SANDRO RIDELLA

Abstract—A lossless distributed rectangular structure, composed of a dielectric layer interposed between two perfectly conductive metallic layers, is considered. This structure is endowed with two ports, whose positions and widths are variable and is connected to the environment through uniform transmission lines. A procedure is exposed to obtain the impedance matrix at the terminals of the transmission lines. The existence of transmission zeros and of filtering properties is demonstrated.

Introduction

TAPERED transmission lines have been investigated both in the lossless and in the RC case by several authors with the use of classical nonuniform line theory [1]–[4].

A more general class of distributed networks has been investigated in recent papers [5]–[7]. These papers are concerned with a three-layer structure in which each layer has uniform thickness and is composed of linear, passive, homogeneous, time-invariant material. Each layer is physically characterized by its magnetic permeability, dielectric constant and electric conductivity. The three layers overlap exactly and are laterally limited by a cylindrical surface of arbitrary shape, perpendicular to the layers. This system interacts with the outside world through N ports placed in the lateral boundary of the wafer. We are interested in the terminal behavior of the system at those N ports. Thus the system studied in [5]–[7] is a generalization of strip lines, nonuniform lines, and various other devices which have been examined in the literature in a rather particular way.

Analysis of the Structure

The analysis of the three-layer wafer described above stems from electromagnetic theory [5]–[7]. The methods of analysis are based on basic techniques, e.g., normal mode methods of field theory which are well known in microwave literature. These methods have been used to investigate particular properties of the three-layer structure. The electromagnetic field inside the structure is represented by expansion in terms of vector eigenfunctions of Maxwell's equations, with properly chosen boundary conditions.

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B. Bianco was with the Istituto Elettrotecnico Nazionale Galileo Ferraris, Torino, Italy. He is now with the Laboratorio per i Circuiti Elettronici, Genoa, Italy.

S. Ridella is with the Istituto di Elettrotecnica, University of Genoa, Genoa, Italy.

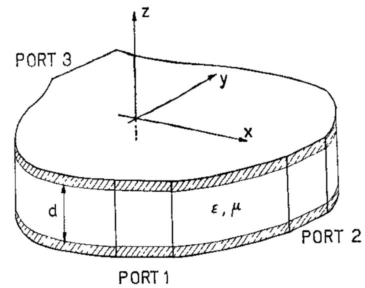

Fig. 1. N-port (here $N=3$) composed of a dielectric layer interposed between two metallic layers.

Such eigenfunctions are broadly divided in two classes: 1) planar ones, only depending on the x and y coordinates, which are parallel to the layers; and 2) depth ones, only depending on the z coordinate (perpendicular to the layers).

It must be remarked that the propagation direction, that is, the direction in which the power flows, lies in the x–y plane; it seems inappropriate to use transverse and longitudinal eigenfunctions instead of planar and depth eigenfunctions, respectively.

The analysis of the terminal behavior of the structure must start from the analysis of the depth properties. In the above quoted papers one can find the general development of this problem.

In this paper, however, we shall consider a three-layer structure (Fig. 1) where the upper and lower layers are perfectly conductive materials and the intermediate one is a lossless dielectric. In technical applications such a structure will be fairly realized by a wafer composed of a dielectric lossless layer interposed between two metallic sheets, obtained, for example, by deposition.

Because of the assumption that the external layers are perfectly conductive, the electromagnetic field in the structure is confined in the intermediate layer. We shall assume that the wafer thickness is so small in comparison with its transverse dimension as to make the effect of field fringing along the boundary practically negligible. Moreover, we shall assume that only the fundamental depth mode [6], [7] is excited; it is characterized by having the electric field parallel to the z axis, and the magnetic field parallel to the x–y plane. These assumptions are quite reasonable for many well-known physical structures, such as lossless uniform and nonuniform lines, microstrip filters, cavities, etc. The terminal behavior of the layered structure can be obtained by solving the following differential equation [5]-[7]:

$$\nabla^2 E_s(x, y) - \gamma^2 E_s(x, y) = 0 \tag{1}$$

where

∇^2 Laplace operator in the x–y plane;
$E_s(x, y)$ electric field (parallel to the z axis);
γ propagation constant, given by

$$\gamma^2 = s^2 \epsilon\mu \tag{2}$$

where $s = \sigma + j\omega$ is the complex frequency and $\epsilon(\mu)$ is the dielectric (magnetic) constant of the intermediate layer.

The boundary conditions for (1) are of the following mixed type. 1) On the part of the boundary not belonging to the ports,

$$\frac{\partial E_s}{\partial n} = 0$$

where n is the inner normal to the boundary. (This condition, which assumes that the boundary of the wafer, in the portions not occupied by the ports, acts as a magnetic wall, derives from that assumption that the wafer is so thin as to make border effects negligible, as said above.) 2) On the part of the boundary belonging to some port, either E_s or $\partial E_s/\partial n$ is assigned, depending on whether an admittance or an impedance point of view is adopted.

The component E_s of the electromagnetic field at port k can be developed as a series of orthogonal functions (the so-called microstrip modes), for instance, as a cosine Fourier series:

$$E_s(s_k) = E_{k,0} + \sum_{r=1}^{\infty} \sqrt{2} E_{k,r} \cos \frac{r\pi s_k}{l_k} \tag{3}$$

where s_k is a running coordinate on port k going from 0 to l_k (kth port width). The Fourier series expansion of the assumed field over the kth port is equivalent to a modal expansion in terms of the modes of a parallel-plate strip line connected to the port.

The next step is to define the voltage and current associated with each port and each mode [which is selected by index r in (3)].

The voltage at port k for a given value of r (*width mode* or microstrip mode) is defined as

$$V_{k,r} = E_{k,r} d \tag{4}$$

where d is the thickness of the dielectric (intermediate) layer. In a similar way one can develop $\partial E_s/\partial n$ at port k:

$$\frac{\partial E_r(s_k)}{\partial n} = H_{k,0} + \sum_{r=1}^{\infty} \sqrt{2} H_{k,r} \cos \frac{r\pi s_k}{l_k}. \tag{5}$$

The current at port k for a given value of r is defined as

$$I_{k,r} = -\frac{H_{k,r} l_k}{s\mu}. \tag{6}$$

These definitions (as well as the boundary conditions) are suggested by the following energy considerations.

1) There is no power entering the system from the part of the boundary not belonging to the ports.

2) The sum of the products VI^* over all width modes at port k will give the power entering the port.

Equation (1), with nonhomogeneous boundary conditions, can be solved by a development in series of open-circuit eigenfunctions defined by

$$\nabla^2 \phi_{n_1 n_2} + \lambda_{n_1 n_2}^2 \phi_{n_1 n_2} = 0 \tag{7}$$

with the boundary condition $\partial \phi_{n_1 n_2}/\partial n = 0$ on the whole border of the structure ($\lambda_{n_1 n_2}^2$ is the eigenvalue associated with the eigenfunction $\phi_{n_1 n_2}$). The eigenfunctions are normalized in such a way that $\int_S \phi_{n_1 n_2}^2 dS = S$, where S is the area of the cross section of the structure.

The knowledge of the whole set of eigenfunctions and related eigenvalues allows the explicit determination of the properties of the system. The assigned field distribution over the ports can be decomposed in a normal mode expansion in terms of the width modes, each of which can be considered separately.

Straightforward calculations [6], [7] give the elements of the Z matrix of the network driven by width mode $r_h(r_k)$ at port $h(k)$:

$$Z_{hk}^{r_h r_k} = \sum_{n_1=0, n_2=0}^{\infty} \beta_{h,n_1,n_2,r_h} \beta_{k,n_1,n_2,r_k} \frac{s\mu d}{(s^2 \epsilon\mu + \lambda_{n_1 n_2}^2) S} \tag{8}$$

where

$$\beta_{h,n_1,n_2,r_k} = \frac{\epsilon r_k}{l_h} \int_0^{l_h} \phi_{n_1 n_2} \cos \frac{r_k \pi s_h}{l_h} ds_h \tag{9}$$

and

$$\epsilon_{r_k} = \begin{cases} 1, & \text{if } r_k = 0 \\ 2, & \text{if } r_k \neq 0. \end{cases}$$

$Z_{hk}^{r_h r_k}$ is the ratio of the voltage of mode r_h at port h to the current of mode r_k at port k when all other currents are zero.

The existence of an impedance matrix is due to linearity. Equation (8) means that the voltage mode r_h at port h can be obtained as a linear combination of all the current modes at the ports. In a similar way one can obtain the Y matrix elements through the short-circuit eigenfunctions [6], [7].

The Z_{TEM} Matrix

In the previous procedure we have assumed an assigned distribution of electromagnetic field at the ports. In order to apply the procedure to real cases, one has to specify by which means such distributions are obtainable. Here we assume that the physical connections between each of the portions of the boundary called ports and the external circuit are uniform strip lines. A general procedure can be developed to obtain the impedance matrix of the N port from the planar eigenfunctions of the structure with the electromagnetic field assigned on the uniform strip lines very far from the ports.

The method is based on the analysis of the transitions among modes in the structure and modes in the lines. We shall describe the procedure for a two-port: the generalization to an N port is straightforward.

We shall define some vector quantities that will be very useful in describing the problem in compact form. We shall indicate $\boldsymbol{I}_1$ and $\boldsymbol{I}_2$ the vectors of the current at port 1 and at port 2: the rth component of these vectors is defined as

$$[\boldsymbol{I}_1(r)] = I_{1,r} \qquad [\boldsymbol{I}_2(r)] = I_{2,r}. \tag{10}$$

The total vector current will be defined as

$$\boldsymbol{I} = \begin{vmatrix} \boldsymbol{I}_1 \\ \boldsymbol{I}_2 \end{vmatrix}. \tag{11}$$

In the same way we shall define the total vector voltage as

$$\boldsymbol{V} = \begin{vmatrix} \boldsymbol{V}_1 \\ \boldsymbol{V}_2 \end{vmatrix} \tag{12}$$

where

$$[\boldsymbol{V}_1(r)] = V_{1,r} \qquad [\boldsymbol{V}_2(r)] = V_{2,r}. \tag{13}$$

From the total vector voltage $\boldsymbol{V}$ one can obtain the fundamental vector voltage $\boldsymbol{E}$ defined as

$$\boldsymbol{E} = \begin{vmatrix} \boldsymbol{V}_1' \\ \boldsymbol{V}_2' \end{vmatrix} \tag{14}$$

where

$$[\boldsymbol{V}_1'(r)] = [\boldsymbol{V}_1(r)\delta_{r0}] \quad \text{and} \quad [\boldsymbol{V}_2'(r)] = [\boldsymbol{V}_2(r)\delta_{r0}]$$

and δ_{r0} is a Kronecker operator. The fundamental vector voltage components are all zero except for two, which are the voltages at port 1 and port 2 for the width mode $r=0$.

It is worth noting that the vectors $\boldsymbol{I}_1$, $\boldsymbol{I}_2$, $\boldsymbol{V}_1$, and $\boldsymbol{V}_2$ have an infinite number of components, that is, r goes from zero to infinity. The use of infinite vectors is quite usual in microwave literature [8]–[10] to allow a compact analysis of the effects of transitions in waveguides and cavities. Now the total voltage vector can be obtained from the total current vector by:

$$\boldsymbol{V} = \boldsymbol{Z}\boldsymbol{I} \tag{15}$$

where $\boldsymbol{Z}$ is the total impedance matrix, that is, a

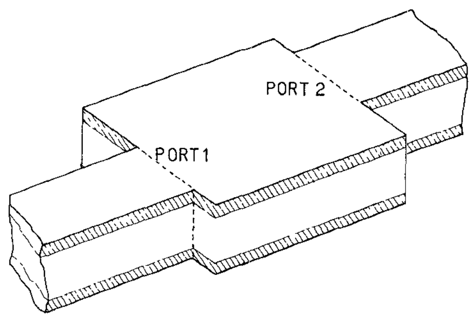

Fig. 2. Rectangular two-port connected to parallel-plate uniform lines.

square matrix defined by

$$\boldsymbol{Z} = \begin{vmatrix} \boldsymbol{Z}_{11} & \boldsymbol{Z}_{12} \\ \boldsymbol{Z}_{21} & \boldsymbol{Z}_{22} \end{vmatrix} \tag{16}$$

where $\boldsymbol{Z}_{11}$, $\boldsymbol{Z}_{12}$, $\boldsymbol{Z}_{21}$, and $\boldsymbol{Z}_{22}$ are square matrices, whose (r_h, r_k) entry is given by

$$[\boldsymbol{Z}_{hk}(r_h, r_k)] = Z_{hk}{}^{r_h r_k}. \tag{17}$$

Equation (15) is derived from the fact that the rth mode voltage at port 1 (or at port 2) is given by a linear combination of all current modes at port 1 and at port 2.

One can obtain from (8) that

$$Z_{hk}{}^{r_h r_k} = Z_{kh}{}^{r_k r_h} \tag{18}$$

that is, the $\boldsymbol{Z}$ matrix is symmetric.

Let us suppose now that the two-port is connected to the environment through two uniform lines. These lines are assumed to have the same physical nature as the given two-port, i.e., they are composed of uniform strips of material of permeability μ, dielectric constant ϵ, thickness d, metallized on the upper and lower faces. Fig. 2 shows a rectangular two-port in such a connection.

The presence of the discontinuities at the ports causes the existence of higher modes both in the uniform lines and in the two-port cavity. These higher modes cannot propagate in the terminations for a wide range of frequency, as when the strip-line widths are very small compared to the wavelength; nevertheless, they are present in the two-port structure.

Then if the length of the uniform lines is sufficiently great as to avoid the coupling of higher modes with the generators or the loads at the terminations, one obtains

$$\boldsymbol{V} = -\boldsymbol{Z}_0\boldsymbol{I} + \boldsymbol{E} \tag{19}$$

where $\boldsymbol{Z}_0$ is the higher modes characteristic impedance matrix composed of four square matrices, two of which are the zero matrix:

$$\boldsymbol{Z}_0 = \begin{vmatrix} \boldsymbol{Z}_{0,1} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{Z}_{0,2} \end{vmatrix}. \tag{20}$$

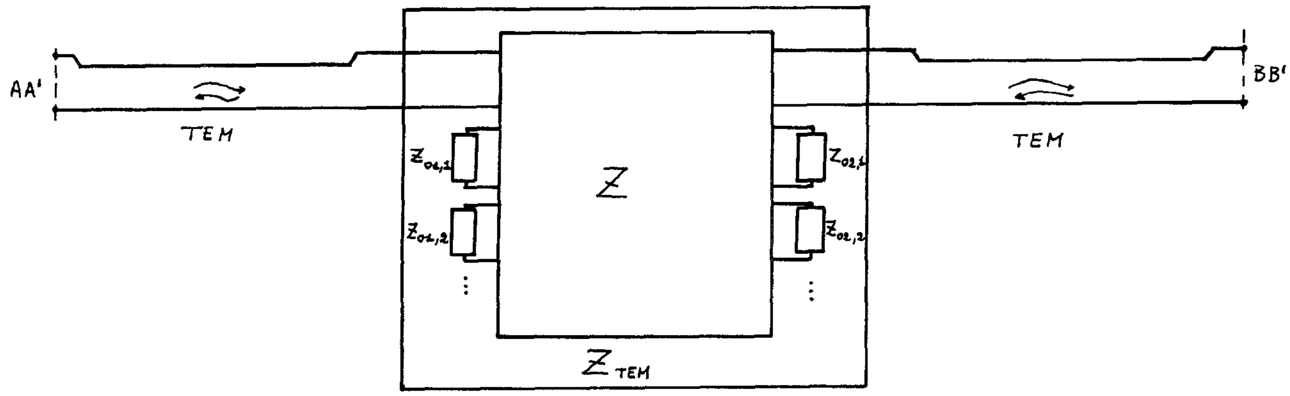

Fig. 3. Equivalent circuit for the Z_{TEM} matrix.

The matrices $\boldsymbol{Z}_{0,1}$ and $\boldsymbol{Z}_{0,2}$ are diagonal matrices defined by

$$[\boldsymbol{Z}_{0,1}(r, r')] = Z_{0,1,r}(1 - \delta_{r0})\delta_{rr'} \quad (21')$$

$$[\boldsymbol{Z}_{0,2}(r, r')] = Z_{0,2,r}(1 - \delta_{r0})\delta_{rr'} \quad (21'')$$

where $(r = 0, 1, 2, \cdots)$

$$Z_{0,1,r} = \frac{s\mu d}{l_1}\left[s^2\epsilon\mu + \left(\frac{r\pi}{l_1}\right)^2\right]^{-1/2} \quad (22')$$

$$Z_{0,2,r} = \frac{s\mu d}{l_2}\left[s^2\epsilon\mu + \left(\frac{r\pi}{l_2}\right)^2\right]^{-1/2} \quad (22'')$$

and l_1 and l_2 are the widths of ports 1 and 2, respectively.

From (15) and (19) one obtains

$$\boldsymbol{I} = (\boldsymbol{Z} + \boldsymbol{Z}_0)^{-1}\boldsymbol{E}. \quad (23)$$

If only the fundamental modes are considered in the total vector current, one obtains

$$\begin{vmatrix} I_{1,0} \\ I_{2,0} \end{vmatrix} = Z_{\text{TEM}}^{-1} \begin{vmatrix} V_{1,0} \\ V_{2,0} \end{vmatrix}. \quad (24)$$

Then Z_{TEM} is defined as the open-circuit impedance matrix of the two-port structure, obtained through elimination of the effects of the attached uniform lines in the impedance matrix at the ports.

From a network point of view the structure of Fig. 2 has the equivalent circuit of Fig. 3.

The Z_{TEM} matrix is the two-port characterization of the structure loaded by the characteristic impedances of the higher modes and driven by a TEM field distribution at the terminals of the uniform lines.

The impedance matrix at the terminals AA' and BB' as shown in Fig. 3 can be obtained by the usual procedures of network theory.

The evaluation of Z_{TEM} requires the inversion of an infinite square matrix. The whole computation can be greatly simplified if one remembers that the definition of Z_{TEM} requires that the uniform lines connected to the ports have very small widths compared with the wavelength. Then it is known that only a few higher modes must be considered [10].

Finally, it is worth noting that the smaller the stripline widths are, the greater the frequency of operation of the system can be without propagation of higher modes in the lines.

The Rectangular Structure

In this section and in the next one we present some results obtained for a structure which is studied rather simply because the open-circuit eigenfunctions are very easily calculated. It is a rectangular structure (Fig. 4) endowed on opposite sides with two ports, whose position and width are variable.

Such a structure is interesting because it is the simplest form of the common uniform line. Still, it shows some unusual features, such as the existence of the transmission zeros, which put into evidence interesting match properties at several frequencies. Hence a full analytical and numerical investigation seems to be worthwhile, not only as an example of a layered structure, but because of possible practical applications.

Straightforward calculations give the Z matrix elements

$$Z_{11}^{rs} = \sqrt{\frac{\mu}{\epsilon}}\,\frac{d}{a}\coth \gamma l \delta_{0r}\delta_{0s} + \frac{2s\mu d}{a} \cdot \sum_{n_2=1}^{\infty} C_1(n_2, r)C_1(n_2, s)\,\frac{\coth l\sqrt{\gamma^2 + \left(\frac{n_2\pi}{a}\right)^2}}{\sqrt{\gamma^2 + \left(\frac{n_2\pi}{a}\right)^2}} \quad (25')$$

$$Z_{22}^{rs} = \sqrt{\frac{\mu}{\epsilon}}\,\frac{d}{a}\coth \gamma l \delta_{0r}\delta_{0s} + \frac{2s\mu d}{a} \cdot \sum_{n_2=1}^{\infty} C_2(n_2, r)C_2(n_2, s)\,\frac{\coth l\sqrt{\gamma^2 + \left(\frac{n_2\pi}{a}\right)^2}}{\sqrt{\gamma^2 + \left(\frac{n_2\pi}{a}\right)}} \quad (25'')$$

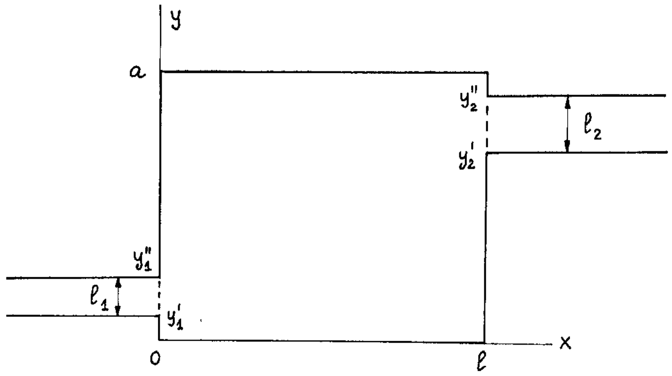

Fig. 4. Rectangular structure.

$$Z_{12}^{rs} = \sqrt{\frac{\mu}{\epsilon}}\,\frac{d}{a}\operatorname{csch}\gamma l\delta_{0r}\delta_{0s} + \frac{2s\mu d}{a} \cdot \sum_{n_2=1}^{\infty} C_1(n_2, r)C_2(n_2, s)\,\frac{\operatorname{csch} l\sqrt{\gamma^2 + \left(\frac{n_2\pi}{a}\right)^2}}{\sqrt{\gamma^2 + \left(\frac{n_2\pi}{a}\right)^2}} \qquad (25''')$$

where ($k = 1, 2$)

$$C_k(n_2, r) = \frac{n_2\epsilon_r}{\pi a l_k} \cdot \frac{(-1)^r \sin\frac{\pi n_2 y_k''}{a} - \sin\frac{\pi n_2 y_k'}{a}}{\left(\frac{n_2}{a}\right)^2 - \left(\frac{r}{l_k}\right)^2}. \qquad (26)$$

From expression (25) (the elements of the Z matrix), one can derive the properties of the rectangular structure, in particular the Z_{TEM} matrix.

An interesting feature of the rectangular structure is the existence of transmission zeros [11]–[14]. These are caused by the propagation of higher modes inside the rectangular structure. At the same time no higher mode must propagate in the strip-line terminations. Thus the cavity width must be much larger than the maximum strip-line width. This simplifies the evaluation of Z_{TEM}. In a first approximation, for very small widths of the ports, one obtains

$$Z_{\text{TEM}\,h,k} \simeq Z_{h,k}^{00}. \qquad (27)$$

This approximation has been used in previous papers [11], [12] to analyze the properties of the structure and to obtain equivalent circuits.

It can be shown that the ordinary stub structures are particular cases of the system under consideration.

Straightforward calculations show that if 1) the widths of the two uniform lines are very small compared to the cavity width, and 2) the cavity length is very small compared to the wavelength, then the structure of Fig. 4 has the equivalent circuit of Fig. 5 for all the frequencies of practical interest.

The equivalent circuit of Fig. 5 is a two-port network composed of three uniform lines, two of which are connected as shunt impedances at the ports. Every line has a propagation constant γ and a characteristic impedance $\sqrt{\mu/\epsilon}(d/l)$; the length of various lines are shown in Fig. 5 when $y_2 > y_1$, where

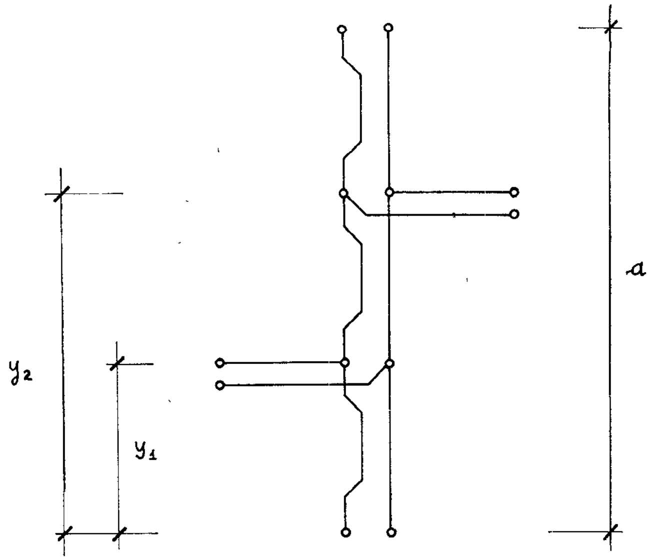

Fig. 5. Generalized stub structure.

$$y_1 = \frac{y_1' + y_1''}{2} \qquad y_2 = \frac{y_2' + y_2''}{2}.$$

This result can be obtained from (25) or (1) directly, noting that, if the length l is very small, $\partial^2 E_s/\partial x^2$ can be completely omitted. Thus the usual equation of uniform lines is obtained. With straightforward imposition of boundary conditions one obtains the equivalent circuit of Fig. 5.

Obviously, from this equivalent circuit one obtains the usual stub structure if ports 1 and 2 are placed symmetrically, particularly, if $y_1 = y_2 = 0$, one realizes that the whole structure is equivalent to an impedance given by the well-known expression

$$Z_{\text{stub}} = \sqrt{\frac{\mu}{\epsilon}}\,\frac{d}{l}\coth(\gamma a). \qquad (28)$$

Then comparison with ordinary stub structures has shown that these are a particular case of the system under consideration.

Equation (27) is correct only in a first approximation. As a second approximation one can consider four elements in every matrix instead of one; that is, in expression (23) one considers Z_{hk}^{01}, Z_{hk}^{10}, and Z_{hk}^{11} beyond Z_{hk}^{00}. This approximation is significant if: 1) only a few width modes inside the cavity can propagate; there exists n_2^* such that $\omega \ll n_2^*\pi/a\sqrt{\epsilon\mu}$ for all the frequencies of practical interest; and 2) the widths of the ports are very small compared to the cavity width, in such a way that $\max(l_1, l_2) \ll (a/n_2^*)$.

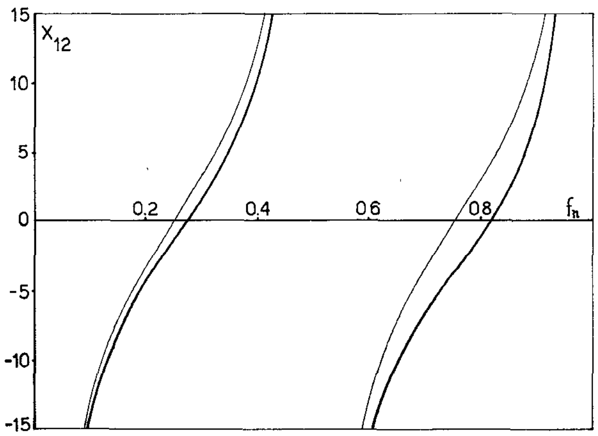

Fig. 6. Comparison with ordinary stub structure.

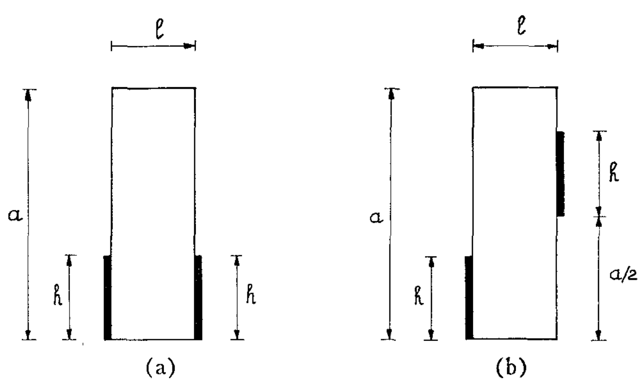

Fig. 7. (a) Stub structure. (b) Modified stub structure.

If these conditions are satisfied, higher modes cannot propagate in the strip-line terminations. The analysis of the transition between cavity and lines can be carried on considering only the fundamental mode and the first higher modes in the terminations. This is quite reasonable because the limitation over the number of cavity-width modes allows a description of the field distribution at the ports through few terms of a cosine Fourier series.

Numerical Analysis

On the background of the theory expressed above, we have prepared a Fortran IV program through which a complete analysis of the structures considered in the previous section can be carried on. It gives both the impedance and scattering matrices (the latter normalized with respect to the characteristic impedances of the lines connected to the ports) once the physical and geometric parameters are specified together with the number of attenuated modes at the ports that one wants to consider.

With the use of this program, a systematic investigation has been made. We present here a brief list of results. The TEM impedance matrix elements are calculated taking into account only the first attenuated port modes.

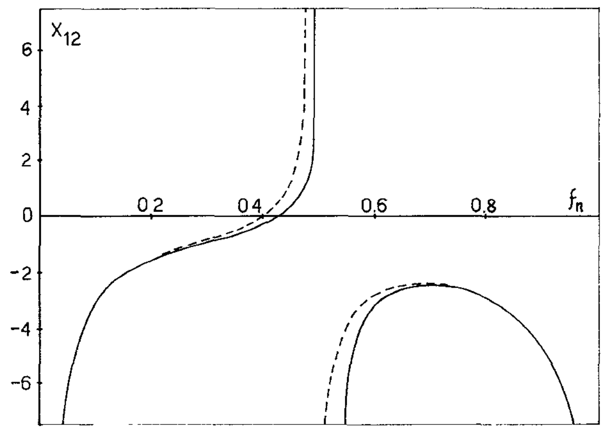

Fig. 8. X_{12} of nonconventional stub structure.

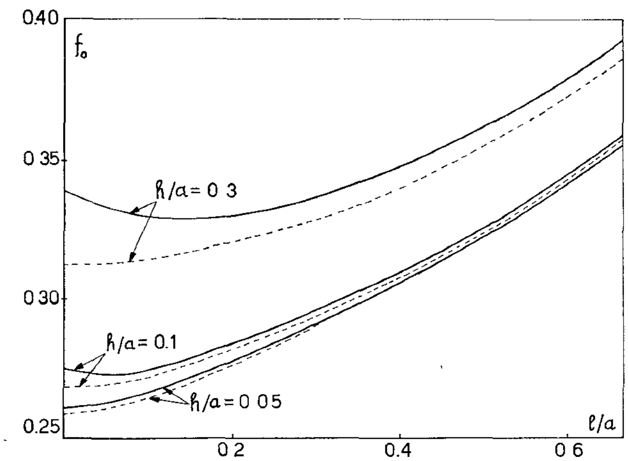

Fig. 9. First transmission zero of the structure of Fig. 7(a).

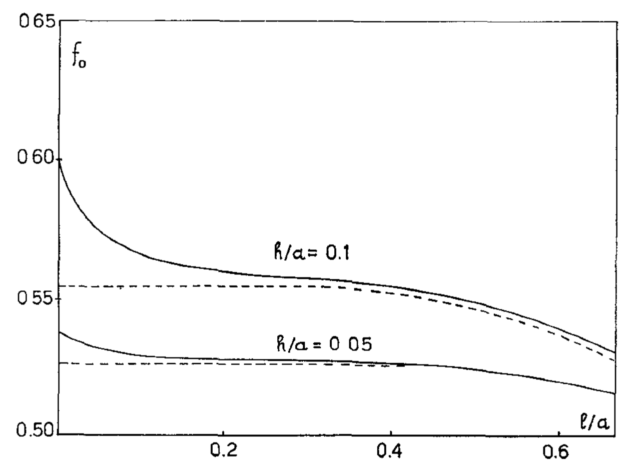

Fig. 10. First transmission zero of the structure of Fig. 7(b).

1) In Fig. 6 one can find the reactance $X_{12\,\text{TEM}}$ (thick line) for the structure of Fig. 7(a), where $h=l=a/10$. This reactance is normalized to $\sqrt{\mu/\epsilon}(d/a)$ and f_n is the normalized frequency, related to the frequency f by $f_n = f\sqrt{\epsilon\mu}a$. In Fig. 6 one can also find the reactance X_{12} (thin line) of the same two-port, consid-

ered as an ordinary stub of length a. $X_{12}{}^{00}$ coincides with $X_{12\ \mathrm{TEM}}$ practically in the scale of the figure.

2) Fig. 8 shows $X_{12\ \mathrm{TEM}}$ (solid line) and $X_{12}{}^{00}$ (dashed line) for the structure of Fig. 7(a), where $h=l=a/2$. Note that, owing to the considerable widths of the ports, this structure by no means can be assimilated to an ordinary stub line, as is apparent from the plots.

3) Fig. 9 shows the normalized frequency f_0 of the lowest transmission zero of the two-port of Fig. 7(a), for various values of h/a, as a function of l/a and calculated from the Z_{TEM} matrix (solid line) and from the Z^{00} matrix (dashed line). It is worth noting in Fig. 9 that for small values of h/a the transmission zeros calculated with both methods are very close and they are significantly different from the frequency of the lowest zero for a simple stub of length a, which is $f_0=0.25$.

4) Fig. 10 shows the normalized frequency of the lowest transmission zero, as in 3) but refers to the structure of Fig. 7(b).

Conclusions

We have shown that by a relatively simple network one can construct very complicated transfer functions. The most interesting feature seems to be the existence of transmission zeros. The results obtained are significant, and they encouraged the authors to develop this investigation in two directions.

One is oriented toward low-frequency applications. The structure acts as bidimensional RC line. The portions of the boundary of the upper and lower layers which belong to the ports are covered by perfectly conductive material. The extension of the results obtained in the lossless case is straightforward.

Another direction is oriented toward high-frequency applications. It has been shown that notch filters of a nonconventional type can be obtained by the use of the structure examined. The effect of the lines connected to the structure on the positions of the first transmission zero has been analyzed for various geometrical dimensions to obtain a notch filter. The results have been plotted with two different degrees of approximation in order to make a comparison with an ordinary stub structure.

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